Penrose Inequality with angular momentum.

$$m^2 + \frac{J^2}{4m^2} \geq \frac{A}{16\pi} + \frac{J^2 4\pi}{A}$$

Natacha Altamirano[1] Sergio Dain[1]

[1]FaMAF-UNC

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Gravitational Collapse

- *Cosmic Censorship*: any gravitational collapse should result in a black hole,
- eventually the state settles down in a stationary state: Kerr black hole (A_0, m_0, J_0)

$$m_0^2 = \frac{A_0}{16\pi} + \frac{J_0^2 4\pi}{A_0}$$

If $\mathcal{M}$ is a Cauchy initial data characterized by (A, m, J) we have

$$m \geq m_0 \quad \text{and} \quad A_0 \geq A\,.$$

$$m^2 \geq m_0^2 = \frac{A_0}{16\pi} + \frac{J_0^2 4\pi}{A_0} \geq \frac{A}{16\pi} + \frac{J^2 4\pi}{A} \quad \text{whenever} \quad A \geq 8\pi J\,.$$

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- For $J = 0$ we recover Penrose inequality

$$m \geq \sqrt{\frac{A}{16\pi}} \tag{1}$$

Huisken-Ilmanen gave a proof using the *Inverse mean curvature flow* equation (1) for assymptotically flat, maximal initial data.

- The restriction

$$A \geq 8\pi J \tag{2}$$

was proved by Dain and Reiris for a minimal surface in an assymptotically flat, axi-symmetric initial data in vacuum.

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Obtained result

$$m^2 \geq \frac{A}{16\pi} + \frac{J^2 4\pi}{A}$$

$$\boxed{m^2 + \frac{J^2}{4m^2} \geq \frac{A}{16\pi} + \frac{J^2 4\pi}{A}}$$

There are two possible ways to reach the result:

- Using the inequalities $A \geq 8\pi J$ and $m^2 \geq \frac{A}{16\pi}$.
- Using the Inverse Mean Curvature flow (IMCF).

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Inverse Mean Curvature Flow

Consists of a flow of 2-surfaces on the initial data which velocity is proportional to the inverse mean curvature in the direction of the outgoing unit normal.

We define

$$m_H = \sqrt{\frac{A}{16\pi}}\left(1 - \frac{1}{16\pi}\int_A H^2 dA\right),$$

with the following properties

- Over a minimal surface with $(H = 0)$ $m_H = \sqrt{\frac{A}{16\pi}}$.
- In infinity $m_H \to m_{ADM}$.
- Over the flow $\dot{m_H} > 0$.

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The functional

Consider the following functional over the IMCF

$$\mathcal{F}(m_H) = m_H^2 + \frac{J^2}{4m_H^2}$$

With the following properties

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So, if we are able to see $\dot{\mathcal{F}} \geq 0$ over the flow we obtain the desired result

$$\boxed{m^2 + \frac{J^2}{4m^2} \geq \frac{A}{16\pi} + \frac{J^2 4\pi}{A}}$$

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The variation of the functional over the flow is

$$\dot{\mathcal{F}} = 2\dot{m_H}\left(m_H - \frac{J^2}{4m_H^3}\right) \overset{?}{\geq} 0$$

As $\dot{m_H} \geq 0$ over the flow then we have to see if

$$m_H^2 \geq \frac{J}{2}$$

But we know

$$m_H^2 \geq m_{H_0}^2 = \frac{A}{16\pi} \geq \frac{8\pi J}{16\pi J} \geq \frac{J}{2}.$$

We used the inequality $A \geq 8\pi J$**!!** This last result shows that $\dot{\mathcal{F}} \geq 0$, in which case

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Inequalities

$$m^2 + \frac{J^2}{4m^2} \geq \frac{A}{16\pi} + \frac{J^2 4\pi}{A}$$

$$f(x) = x + \frac{J^2}{4x}$$

$$f(m^2) = m^2 + \frac{J^2}{4m^2} \quad \text{and} \quad f\left(\frac{A}{16\pi}\right) = \frac{A}{16\pi} + \frac{J^2 4\pi}{A} \qquad (3)$$

The function is increasing whenever $x \geq \frac{J}{2}$. If $x = \frac{A}{16\pi}$ then $f(x)$ will be increasing when $A \geq 8\pi J$. In addition if $m^2 \geq \frac{A}{16\pi}$ and the fuction is increasing we have

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Inequalities

$$\boxed{m^2 + \frac{J^2}{4m^2} \geq \frac{A}{16\pi} + \frac{J^2 4\pi}{A}}$$

$$f(x) = x + \frac{J^2}{4x}$$

$$f(m^2) = m^2 + \frac{J^2}{4m^2} \quad \text{and} \quad f\left(\frac{A}{16\pi}\right) = \frac{A}{16\pi} + \frac{J^2 4\pi}{A} \tag{3}$$

The function is increasing whenever $x \geq \frac{J}{2}$. If $x = \frac{A}{16\pi}$ then $f(x)$ will be increasing when $A \geq 8\pi J$. In addition if $m^2 \geq \frac{A}{16\pi}$ and the fuction is increasing we have

$$m^2 \geq \frac{A}{16\pi} \quad \Rightarrow \quad f(m^2) \geq f\left(\frac{A}{16\pi}\right)$$

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Final Remarks

In the context of gravitational collapse we were able to show a new inequality

$$m^2 + \frac{J^2}{4m^2} \geq \frac{A}{16\pi} + \frac{J^2 4\pi}{A}$$

- Using the inequalities $A \geq 8\pi J$ and $m^2 \geq \frac{A}{16\pi}$.
- Using the Inverse Mean Curvature flow + $A \geq 8\pi J$.

This inequality is weaker than the Generalized Penrose inequality

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Thank you!